Constructing collateralized (OIS) USD discounting curve and USD 1M forward curve

1. USD OIS discount curve

For the construction of the OIS discount curve, we will assume that the OIS trade is fully cash collateralized (to mitigate counterparty credit risk) and collateral rate on cash equals the overnight rate. OIS DFs will be determined by using quoted overnight indexed swaps (OIS) rates. As part of OIS trade, fixed coupon is exchanged for a daily compounded overnight rate, where the dates of the two payments typically coincide. Hence, between two payment dates T_{l-1} and T_l the floating leg pays (assume USD 1notional and OIS trade fully collateralized):

$$\prod_{s=T_{l-1}}^{T_l}(1+\delta_s * c(s)) - 1 \qquad \text{………. Equation (1)}$$

With δ_s is the year fraction and $c(s)$ is the collateral rate at time s, by means of Equation (1) we approximate the figure for daily compounded floating leg amount.

In non-stochastic compounded interest rate setup, the summation of PV of floating side individual CFs for the OIS can be represented in line with the Equation (2) (Explained in Annex to IRCC1) as follows:

$$\sum_{k=1}^{k=N} \{D(t,T_{k-1}) - D(t,T_k)\} \qquad \text{………. Equation (2)}$$

In the process of summation, in between terms will get eliminated and PV for the floating side of the OIS will be in the following form:

$$D(t,T_0) - D(t,T_N) \qquad \text{………. Equation (3)}$$

In terms of fixed leg of the OIS, Where $S(t,T_N)$ is the time-t fair swap rate for an OIS of length T_N; $\delta_{l-1,l}^{fi}$ is the fixed leg year fraction between nodes $l-1$ and l, PV of the fixed leg side CFs can be represented as follows:

$$S(t,T_N)\sum_{l=1}^{N}\delta_{l-1,l}^{fi}\,D(t,T_l) \qquad \text{………. Equation (4)}$$

For par OIS, the fixed (Equation (4)) and floating legs (Equation (3)) of the OIS will get equated as follows:

$$S(t,T_N)\sum_{l=1}^{N}\delta_{l-1,l}^{fi}\,D(t,T_l) = D(t,T_0) - D(t,T_N) \qquad \text{………. Equation (5)}$$

By Mathematical induction, term structure of sequential DFs can be derived as follows:

$$D(t,T_l) = \frac{D(t,T_0) - S(t,T_l)\sum_{i=1}^{l-1}\delta_{i-1,i}^{fi}D(t,T_i)}{1+S(t,T_l)\delta_{l-1,l}^{fi}} \qquad \text{………. Equation (6)}$$

Basis the preceding equation a proper discount curve can be obtained by appropriate choice of interpolation technique.

2. USD 1M Libor based forward curve

To begin with we consider a 1M TTM USD par swap and equate its fixed and float leg to compute the USD 1M implied swap rate as follows:

$$L^{1m}(0,1m) = \frac{\delta_{0,1m}^{fi}}{\delta_{0,1m}^{1m}}[C(0,1m)] \qquad \text{………. Equation (7)}$$

In Equation (1), $C(0,1m)$ denotes $ denominated USD 1M par swap rate; $\delta_{0,1m}^{1m}$ is the USD 1 month float leg year fraction; $\delta_{0,1m}^{fi}$ is the USD 1-month fixed leg year fraction and $[L^{1m}(0,1m)]$ is the non-stochastic forward rate for the USD 1M Libor rate.

For longer market quoted USD IRS (≤12M), we can write a generic par swap equation for the fixed (LHS of Equation (8)) and float (RHS of Equation (8)) legs as described below, and we can extract the USD 1M Libor forward rates for maturities ≤12M through this equation:

$$C(t,T_N)\sum_{k=1}^{N}\delta_{k-1,k}^{fi}D(t,T_k) = \sum_{k=1}^{N}\delta_{k-1,k}^{1m}[L^{1m}(T_{k-1},T_k)]D(t,T_k) \qquad \text{………. Equation (8)}$$

Given that the par swap rate (i.e. C(t,T_N)) are market quoted and the DFs can be calculated through generic DF formulas, the first 12 non-stochastic USD 1M forward rates i.e. [L^1m (0m,1m)], [L^1m (1m,2m)] …., [L^1m (11m,12m)] can be solved using matrix algebra with respect to Equation (8).

Given that these forward rates are with 1M discretion, it obviates the need to interpolate. Then, we compute the implied tenor basis spreads (TS) for maturities 3M, 6M, 9M and 12M by using the derived short-end of the market quoted USD 3M forward curve. The USD 1M forward curve in a non-stochastic setup is derived based on equating the USD 1M leg (TS adjusted, LHS of Equation (9) below) against the USD 3M leg (RHS of Equation (9) below) under the USD 1M vs 3M basis swap trade.

$$\sum_{k=1}^{N}\delta_{k-1,k}^{1m}\{[L^{1m}(T_{k-1},T_k)] + TS(t,T_N)\}D(t,T_k) = \sum_{n=1}^{N}\delta_{n-1,n}^{3m}[L^{3m}(T_{n-1},T_n)]D(t,T_n) \text{...Equation (9)}$$

While running this USD 1M forward curve generation procedure, interpolation of the basis spreads will be necessary because of large set of unknown (to be precise 3 times) [L^1m (T_(k-1),T_k)]. Since in a typical USD IRS, fixed leg pays interest semiannually and here the USD 1M float leg will pay the interest on a monthly basis, we won’t be able to solve the system of equation, to overcome this issue we will assume that the USD 1M forward rates will be piecewise flat in running pairs, which in turn implies bringing in assumption such as:

$$[L^{1m}(12m,13m)] = [L^{1m}(13m,14m)] = [L^{1m}(14m,15m)];$$

$$[L^{1m}(15m,16m)] = [L^{1m}(16m,17m)] = [L^{1m}(17m,18m)];$$

$$[L^{1m}(18m,19m)] = [L^{1m}(19m,20m)] = [L^{1m}(20m,21m)];$$

$$\vdots$$

$$\vdots$$

For the first 12 months, we will have to adjust (reduce) the following sum in the 3M leg of the USD basis swap:

$$A = \sum_{i=1m}^{12m} \delta_{i-1m,i}^{1m} \left[L^{1m}(i-1m,i)\right] D(0,i) \qquad \text{...Equation (10)}$$

It will help remove the summative effect of first 12 USD 1M forward rates (already covered by Equation (8)) from the USD 3M implied forward rates, after adjusting for A the preceding 1M vs 3M basis swap equality i.e. Equation (9) can be solved for array of USD 1M forward.